

Photon as a Vector Goldstone Boson: Nonlinear σ Model for QED

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We show that QED in the Coulomb gauge can be considered as a low energy linear approximation of a non-linear σ -type model where the photon emerges as a vector Goldstone boson related to the spontaneous breakdown of Lorentz symmetry down to its spatial rotation subgroup at some high scale M . Starting with a general massive vector field theory one naturally arrives at this model if the pure spin-1 value for the vector field $A_\mu(x)$ provided by the Lorentz condition $\partial_\mu A_\mu(x) = 0$ is required. The model coincides with conventional QED in the Coulomb gauge for $M \rightarrow \infty$ and generates a very particular form for the Lorentz and CPT symmetry breaking terms, which are suppressed by powers of M .

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INTRODUCTION

In recent years there has been considerable interest in the breakdown of the Lorentz invariance, as a phenomenological possibility [1] in the context of various quantum field theories as well as modified gravity and string theories[2, 3, 4, 5]. The existence of Lorentz violation leads to a plethora of new high energy effects [1, 6] with interesting implications for neutrino experiments as well as high energy cosmic ray phenomena. Furthermore, for particular parameterizations of Lorentz violation, these high energy effects can also lead to bounds on the strengths of these effects.

In this note we discuss Lorentz violating effects within a new formulation of Quantum Electrodynamics (QED), where the familiar QED in Coulomb gauge emerges as an effective low energy Lagrangian in a theory where the masslessness of photon is related to spontaneous breakdown of the Lorentz symmetry (SBLS). Lorentz symmetry and its spontaneous breakdown is an old idea[7] and have been considered in many different contexts[8], particularly, in the generation of the internal symmetries observed in particle physics, although many formulations of the idea look contradictory (see for some recent criticism [2, 9]). Our point is that an adequate formulation of the SBLS should be related to a fundamental vector field Lagrangian by itself rather than an effective field theory framework containing a (finite or infinite) set of the primary fermion interactions where the vector fields appear as the auxiliary fields for each of these fermion bilinears¹.

Actually, the symmetry structure of the existing theories, like as QED or Yang-Mills Lagrangians, seems to be well consistent with such a point of view [11]. The vector field gauge-type transformation, say for QED, of the form

$$A_\mu(x) \rightarrow A_\mu(x) + n_\mu \quad (\mu = 0, 1, 2, 3), \quad (1)$$

can be identified by itself as the pure SBLS transformation with vector field $A_\mu(x)$ developing some constant background value n_μ ². The point is, however, this Lorentz symmetry breaking does not manifest itself in any physical way, due to the fact that an invariance of the QED under the transformation (1) leads to the conversion of the SBLS into gauge degrees of freedom of the massless photon. This in essence is what we call the non-observability of the SBLS of type (1). In this connection it was recently shown [11] that gauge theories, both Abelian and non-Abelian, can be obtained from the requirement of the physical non-observability of the SBLS (1), caused by the Goldstonic nature of vector fields, rather than from the standard gauge principle.

It is instructive here to compare this QED case with the free massless (pseudo-) scalar triplet theory ($\partial^2 \phi^i = 0$), which is invariant under a similar spontaneous symmetry breaking transformation

$$\phi^i(x) \rightarrow \phi^i(x) + c^i \quad (i = 1, 2, 3), \quad (2)$$

where the c^i are arbitrary constants. Again this symmetry transformation corresponds to zero-mass excitations

¹ There is no need, in essence, for the physical SBLS to generate "composite" vector bosons (related with fermion bilinears) which could mediate the gauge-type binding interactions in Abelian or non-Abelian theories[7, 8]. This conclusion most clearly follows from the proper lattice formulation [10] where Lorentz invariance is explicitly broken at the very beginning.

² Remarkably, it was argued a long time ago [12] that just the invariance of the QED under the transformations (1) with a gauge function linear in the co-ordinates ($A_\mu \rightarrow A_\mu + \partial_\mu \omega$, $\omega = n_\mu x^\mu$) implies that the theory contains a genuine zero mass vector particle.

of the vacuum, which might be identified (in some approximation) with physical pions. However, in marked contrast with the vector field case (1) where renormalizable interactions of the form $\bar{\psi}\gamma_\mu\psi A^\mu$ are invariant under the transformation (1) (accompanied by $\psi \rightarrow e^{in\cdot x}\psi$), the scalar field theory invariant under (2) ends up being a trivial theory. The only way to have it as an interacting theory is to add additional states such as the σ field as in the famous σ -model[13] of Gell-Mann and Levy.

A question then is whether one can have an alternative formulation of QED where the symmetry transformation in (1) is realized in a manner analogous to the σ -model and, if so, what kind of new physics it leads to in addition to the familiar successes of QED.

It is quite clear that the simplest way to retain the explicitly covariant form of the vector Goldstone boson transformation (1), is to enlarge the existing Minkowskian space-time to higher dimensions with our physical world assumed to be located on a three-dimensional Brane embedded in the high-dimensional bulk. However, while technically it is quite possible to start, say, with the spontaneous breakdown of the 5-dimensional Lorentz symmetry $SO(1,4) \rightarrow SO(1,3)$ [14] to generate an ordinary 4-dimensional vector Goldstone vector field $A_\mu(x)$, a serious problem for such theories is how to achieve the localization of this field on the flat Brane associated with our world [16]. We therefore take an alternative path: we start with a general massive vector field theory in an ordinary 4-dimensional space-time. The only restriction imposed is the requirement that the four-vector A_μ in order to describe a spin-1 particle, must satisfy the Lorentz condition³

$$\partial_\mu A^\mu(x) = 0 \quad (3)$$

In this connection, it seems important to note that we are dealing further with just the physical vector field condensation rather than a condensation of the scalar component in the 4-vector $A_\mu(x)$, as might occur in the general case when the supplementary Lorentz condition (3) is not imposed. We show that this leads to a non-linear σ -type model for QED, where the photon emerges as a vector Goldstone boson related to the spontaneous breakdown of Lorentz symmetry down to its spatial rotation subgroup $SO(1,3) \rightarrow SO(3)$ at some high scale M . The model appears to coincide with ordinary QED taken in the Coulomb gauge in the limit where the scale M goes to infinity. For finite values of M , there appear an infinite number of nonlinear photon interaction and self-interaction terms properly suppressed by powers of M . These terms violate Lorentz invariance and could have interesting implications for physics.

THE SPIN-1 VECTOR FIELDS AND PHYSICAL SBLs

Let us consider a simple Lagrangian for the neutral vector field $A_\mu(x)$ and one fermion $\psi(x)$ with dimensionless coupling constants (λ and e)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{2}A_\mu^2 - \frac{\lambda}{4}(A_\mu^2)^2 + \bar{\psi}(i\gamma\partial - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi \quad (4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ denotes the field strength tensor for the vector field $A_\mu = (A_0, A_i)$ (and we denote $A_\mu A^\mu \equiv A_\mu^2$ and use similar shorthand notation e.g. $(\partial_\mu A_i)^2 \equiv \partial_\mu A_i \partial^\mu A_i$ etc. later). The free part of the Lagrangian is taken in the standard form so that in this case the Lorentz condition (3) automatically follows from the equation for the vector field A_μ . The λA^4 term is added to implement the spontaneous breakdown of Lorentz symmetry $SO(1,3)$ down to the $SO(3)$ or $SO(1,2)$ for $\mu^2 > 0$ and $\mu^2 < 0$, respectively. Note that contribution of the form A^4 (and higher)⁴ naturally arises in string theories[1, 15].

Writing down the equation of motion for vector and fermion fields

$$\partial^2 A_\mu - \partial_\mu \partial^\nu A_\nu + \mu^2 A_\mu - \lambda A_\mu A_\nu^2 - e\bar{\psi}\gamma_\mu\psi = 0 \quad (5)$$

$$(i\gamma\partial - m)\psi - eA^\mu\gamma_\mu\psi = 0 \quad (6)$$

taking then the 4-divergence of Eq.(5), and requiring that the Lorentz condition (3) be fulfilled, one comes to the equation

$$\lambda\partial_\nu(A^\mu A_\mu) = 0 \quad (7)$$

which should be satisfied identically. Otherwise, it would represent by itself one more supplementary condition (in addition to the equations of motions and Lorentz condition) implying that the field A_μ has fewer degrees of freedom than is needed for describing all its three possible spin states. This is definitely inadmissible. Furthermore a solution of the form $\lambda = 0$ is also not acceptable for $\mu^2 < 0$ since in this case, the Hamiltonian for the theory has no lower bound. Thus the only solution to Eq. (7) for the physical massive vector field corresponds generally to the case

$$\lambda \neq 0, \quad A_\mu^2 = M^2 \quad (8)$$

where $M^2 = \frac{\mu^2}{\lambda}$ stands for some arbitrary constant parameter with dimensionality of mass squared. The gen-

³ This supplementary condition is in fact imposed as an off-shell constraint, independent of its equation of motion[17].

⁴ In fact, one might add one more term of the form $A_\mu A^\nu \partial_\nu A^\mu$, making the neutral vector field Lagrangian (4) to be the most general parity-conserving theory with only terms of dimension ≤ 4 . However, in the ground state of interest here, this extra term vanishes and leads to the same physics (see below).

eral Lagrangian (4) now takes the form

$$\mathcal{L}_{SBLs} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma\partial + m)\psi - eA^\mu\bar{\psi}\gamma_\mu\psi + \text{const}, \quad (9)$$

with the important constraint

$$A_0^2 - A_i^2 = M^2,$$

from which it will be obviously noticed that the vector field A_μ appears massless, while its vev leads to the actual SBLs. We have obtained in fact the nonlinear σ -type model for QED which we now expand in more detail. Remarkably, there is no other solution to our basic equation (7) inspired solely by the spin-1 requirement for massive vector field (3). Furthermore, the condition (8) arises regardless of the sign of μ^2 leading to the global minimum of the theory (given by constant term in \mathcal{L}_{SBLs}) which locates lower than the case where the vector field has zero vev. However, for the right sign mass term case ($\mu^2 > 0$) taken in the starting Lagrangian (4) the Lorentz symmetry always breaks down to its spatial rotation subgroup $SO(1,3) \rightarrow SO(3)$. This is the main result of the paper, whose implications we study now⁵.

NONLINEAR σ MODEL FOR QED

The above considerations allow us to argue that the spin-1 vector field A_μ can selfconsistently be presented in the Lorentz symmetry phase as the massive vector field mediating the fermion (and any other matter) interactions in the framework of the massive QED, or, conversely, in the physical SBLs phase (9) as the basic condensed field producing massless Goldstone states which then are identified with physical photons.

Taking the characteristic SBLs parameter M^2 positive ($M^2 > 0$) one comes to the breakdown of the Lorentz symmetry to its spatial rotation subgroup with the vector field space-components A_i ($i = 1, 2, 3$) as the Goldstone fields. Their Lagrangian immediately follows from

Eq.(9) which after using the Lorentz condition (3) and elimination of the vector field time-component A_0 looks like

$$\begin{aligned} \mathcal{L}_{QED\sigma} &= -\frac{1}{2}(\partial_\mu A_\nu)^2 + \bar{\psi}(i\gamma\partial + m)\psi - eA^\mu\bar{\psi}\gamma_\mu\psi \\ &= \frac{1}{2}(\partial_\mu A_i)^2 - \frac{1}{2}\frac{(A_i\partial_\mu A_i)^2}{M^2 + A_i^2} + \\ &\quad + \bar{\psi}(i\gamma\partial + m)\psi + eA_i\bar{\psi}\gamma_i\psi \\ &\quad - e\sqrt{M^2 + A_i^2}\bar{\psi}\gamma_0\psi \end{aligned} \quad (10)$$

We now expand the newly appeared terms in powers of $\frac{A_i^2}{M^2}$ and also make the appropriate redefinition of fermion field ψ according to

$$\psi \rightarrow e^{ieMx_0}\psi \quad (11)$$

so that the mass-type term $eM\bar{\psi}\gamma_0\psi$, appearing from the expansion of the fermion current time-component interaction in the Lagrangian (10) will be exactly cancelled by an analogous term stemming now from the fermion kinetic term. After this redefinition, and collecting the linear and nonlinear (in the A_i fields) terms separately, we arrive at the Lagrangian

$$\begin{aligned} \mathcal{L}_{QED\sigma} &= \frac{1}{2}(\partial_\mu A_i)^2 + \bar{\psi}(i\gamma\partial + m)\psi + eA_i\bar{\psi}\gamma_i\psi - (12) \\ &\quad - \frac{1}{2}\frac{(A_i\partial_\mu A_i)^2}{M^2} \left(1 - \frac{A_i^2}{M^2} + \dots\right) \\ &\quad - e\frac{A_i^2}{2M} \left(1 - \frac{A_i^2}{4M^2} \dots\right) \bar{\psi}\gamma_0\psi \end{aligned}$$

where we have retained the former notation for the fermion ψ and omitted the higher nonlinear terms for photon. Additionally, the Lorentz condition for the spin-1 vector field (3) now reads as follows:

$$\partial_i A_i - \frac{A_i\partial_0 A_i}{M} \left(1 - \frac{A_i^2}{2M^2} + \dots\right) = 0 \quad (13)$$

The Lagrangian (12) together with a modified Lorentz condition (13) completes the σ model construction for quantum electrodynamics. We will call this $QED\sigma$. The model contains only two independent (and approximately transverse) vector Goldstone boson modes which are identified with the physical photon, and in the limit $M \rightarrow \infty$ is indistinguishable from conventional QED taken in the Coulomb gauge. In this limit the 3-dimensional analog of the “goldstonic” gauge transformations (1) accompanied by the proper phase transformation of fermion

$$A_i(x) \rightarrow A_i(x) + n_i, \quad \psi \rightarrow e^{-ien_i x_i} \psi \quad (i = 1, 2, 3) \quad (14)$$

emerges as an exact symmetry of the Lagrangian in Eq. (12), as one would expect in the pure Goldstonic phase.

While $QED\sigma$ coincides with the conventional QED in Coulomb gauge in the limit of $M \rightarrow \infty$, it differs

⁵ Note that some models of QED with a nonlinear condition $A_0^2 - A_i^2 = M^2$ has been previously considered [15, 18, 19]. In refs. [18, 19], this condition was used as a pure gauge choice whereas in our case it is a dynamical constraint (stemming from the spin-1 requirement for vector field) since our basic massive vector field Lagrangian does not have gauge invariance. In Ref.[15], this condition appeared as a symmetry breaking condition in the string-inspired models (called “bumblebee models”) with an effective negative-sign mass square term for vector field. A crucial difference between the present work and the work of ref. [15] is that in our case the dynamical spin-1 requirement (3) leads to the constraint in Eq.(8) for vector field regardless of the sign of its mass-term. Apart from the fact that for the case of right (positive) sign for μ^2 one has the global minimum of the theory for the SBLs of type $SO(1,3) \rightarrow SO(3)$, this requirement also excludes any ghost like modes in the model. As a result, our model is fundamentally different, with a very different effective Lagrangian at low energies.

from the conventional QED in Coulomb gauge in several ways. First, apart from an ordinary photon-fermion coupling, our model generically includes an infinite number of nonlinear photon interaction and self-interaction terms which become active at high energies comparable to the SBLS scale M . Second and more important, the nonlinear photon interaction terms in the Lagrangian Eq. (12) break Lorentz invariance in a very specific way depending only on a single parameter M unlike many recent parameterizations of Lorentz breaking which involve more than one new parameter. Furthermore all the non-linear photon-fermion (photon-matter in general) interaction terms are C, CP and CPT non-invariant as well. This should have interesting implications for particle physics and cosmology, such as high precision measurements involving atomic systems, breaking of C and CP invariance in electromagnetic processes, extra contribution to neutral mesons oscillations and, especially, the implications of CPT-violating effects on the matter-antimatter asymmetry in the early universe. We will pursue these implications in a separate publication. One immediate point to note is that the dispersion formula for light propagation still remains the same (i.e. $\omega_k^2 - |\vec{k}|^2 = 0$).

CONCLUSION

To summarize, we have started with the observation[11] that the gauge-type transformations (1) with a gauge function linear in the co-ordinates can be treated as the transformations of the spontaneously broken Lorentz symmetry, whose pure Goldstonic phase is presumably realized in the form of the known QED. Exploring this point of view and starting from the general massive vector field theory, we have constructed a full theoretical framework for the physical SBLS including its Higgs phase as well, in terms of the properly formulated nonlinear σ -type model. For the first time we have proposed a pure fundamental Lagrangian formulation without referring to the effective four-fermion interaction ansatz dating back to the pioneering work of Bjorken [7].

In this connection, one might conclude that the whole non-linear Lagrangian $\mathcal{L}_{QED\sigma}$ (12), with a massless photon provided by the spontaneous breakdown of Lorentz invariance, is in some sense a more fundamental theory of electromagnetic interactions than the usual QED⁶. This

theory coinciding with quantum electrodynamics at low energies happens to generically predict striking new phenomena beyond conventional QED at high energies comparable to the SBLS scale M - an infinite number of nonlinear photon-photon and photon-matter interactions which explicitly break relativistic invariance, and C, CP and CPT symmetry.

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⁶ Indeed this origin for the masslessness of the photon seems to be more general and deep than the usually postulated gauge symmetry. Despite the essentially non-renormalisable character of the Lagrangian $\mathcal{L}_{QED\sigma}$ (12), one does not expect the radiative corrections to generate a mass for the photon; otherwise one would have to admit that the radiative corrections lead to a breakdown of the original Lorentz symmetry in the starting Lagrangian (4)

or (9), which is hardly imaginable.

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